**Topic**: Chain rule with quotient rule

**Question**: Use quotient rule and chain rule to find the derivative.

$$f(x) = \frac{\left(2x^2 + 1\right)^3}{x}$$

**Answer choices**:

A $f'(x) = 12x\left(2x^2 + 1\right)^2$

B $f'(x) = \dfrac{\left(2x^2 + 1\right)^2\left(10x^2 - 1\right)}{x^2}$

C $f'(x) = \dfrac{1}{12x\left(2x^2 + 1\right)^2}$

D $f'(x) = \dfrac{1 - 10x^2}{\left(2x^2 + 1\right)^4}$

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**Solution**: B

Since

$$f(x) = \frac{\left(2x^2 + 1\right)^3}{x}$$

is the quotient of two functions of $x$, we'll need to use the quotient rule to differentiate $f(x)$.

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{\left[h(x)\right]^2}$$

In this case, let $g(x) = \left(2x^2 + 1\right)^3$ and $h(x) = x$. Next, we need to find the derivatives $g'(x)$ and $h'(x)$, so that we can use the quotient rule formula above. Finding $h'(x)$ is straightforward (since $h(x) = x$, we know that $h'(x) = 1$). However, note that $g(x)$ is a composite function: if we let $j(u) = u^3$ and $u = k(x) = 2x^2 + 1$, we'll find that $g(x) = j(k(x))$.

Therefore, we'll need to use the chain rule to differentiate $g(x)$.

$$g(x) = j(k(x))$$

$$g'(x) = j'(k(x)) \cdot k'(x)$$

Let's find the derivatives of $j$ and $k$ to substitute in.

$$j(u) = u^3$$

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$$j'(u) = 3u^2$$

$$j'(k(x)) = 3\left(2x^2+1\right)^2$$

and

$$k(x) = 2x^2 + 1$$

$$k'(x) = 2(2x) + 0 = 4x$$

Therefore,

$$g'(x) = j'(k(x)) \cdot k'(x) = 3\left(2x^2+1\right)^2 \cdot 4x = 12x\left(2x^2+1\right)^2$$

Now, we can use the information we've found to substitute into our original formula and find $f'(x)$.

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{\left[h(x)\right]^2}$$

$$f'(x) = \frac{12x\left(2x^2+1\right)^2 \cdot x - \left(2x^2+1\right)^3 \cdot 1}{(x)^2}$$

$$f'(x) = \frac{12x^2\left(2x^2+1\right)^2 - \left(2x^2+1\right)^3}{x^2}$$

$$f'(x) = \frac{\left(2x^2+1\right)^2\left[12x^2 - \left(2x^2+1\right)\right]}{x^2}$$

$$f'(x) = \frac{\left(2x^2+1\right)^2\left(12x^2 - 2x^2 - 1\right)}{x^2}$$

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$$f'(x) = \frac{\left(2x^2+1\right)^2\left(10x^2-1\right)}{x^2}$$

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**Topic**: Chain rule with quotient rule

**Question**: Use quotient rule and chain rule to find the derivative.

$$y = \frac{\sin 3x}{\cos x^2}$$

**Answer choices**:

A $$\frac{dy}{dx} = \frac{-2x \sin 3x \sin x^2 - 3 \cos 3x \cos x^2}{\sin^2 3x}$$

B $$\frac{dy}{dx} = -\frac{3 \cos 3x}{2x \sin x^2}$$

C $$\frac{dy}{dx} = \frac{3 \cos 3x \cos x^2 + 2x \sin 3x \sin x^2}{\cos^2 x^2}$$

D $$\frac{dy}{dx} = -3 \cos 3x \cos x^2 + 2x \sin 3x \sin x^2$$

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**Solution**: C

First, note that

$$y = \frac{\sin 3x}{\cos x^2}$$

is the quotient of two functions of $x$. If we let $v = \sin 3x$ and $w = \cos x^2$, then $y = v/w$. Therefore, we'll need to use the quotient rule to differentiate the function.

$$y = \frac{v}{w}$$

$$\frac{dy}{dx} = \frac{w \cdot \frac{dv}{dx} - v \cdot \frac{dw}{dx}}{w^2}$$

To do so, we'll need to find $dv/dx$ and $dw/dx$. Note that $v$ is a composite function ("function of a function"): $v = f(u) = f(g(x))$ where $f(u) = \sin u$ (the "outer function") and $u = g(x) = 3x$ (the "inner function"). Therefore, finding the derivative $dv/dx$ involves using chain rule.

$$\frac{dv}{dx} = \frac{dv}{du} \cdot \frac{du}{dx}$$

$$\frac{dv}{dx} = \cos u \cdot 3$$

$$\frac{dv}{dx} = 3 \cos 3x$$

Similarly, $w$ is also a composite function ($w = \cos z$ where $z = x^2$), and so finding $dw/dx$ also involves the chain rule.

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$$\frac{dw}{dx} = \frac{dw}{dz} \cdot \frac{dz}{dx}$$

$$\frac{dw}{dx} = -\sin z \cdot 2x$$

$$\frac{dw}{dx} = -2x \sin x^2$$

Now we have the pieces we need to find $dy/dx$.

$$\frac{dy}{dx} = \frac{w \cdot \frac{dv}{dx} - v \cdot \frac{dw}{dx}}{w^2}$$

$$\frac{dy}{dx} = \frac{\cos x^2 \cdot 3\cos 3x - \sin 3x \cdot -2x \sin x^2}{\left(\cos x^2\right)^2}$$

$$\frac{dy}{dx} = \frac{3\cos 3x \cos x^2 + 2x \sin 3x \sin x^2}{\cos^2 x^2}$$

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**Topic**: Chain rule with quotient rule

**Question**: Use quotient rule and chain rule to find the derivative.

$$f(x) = \frac{\ln 2x}{x^2}$$

**Answer choices**:

A $$f'(x) = \frac{4x \ln 2x - x}{2(\ln 2x)^2}$$

B $$f'(x) = \frac{1 - 4 \ln 2x}{2x^3}$$

C $$f'(x) = \frac{2x \ln 2x - x}{(\ln 2x)^2}$$

D $$f'(x) = \frac{1 - 2 \ln 2x}{x^3}$$

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**Solution**: D

Since

$$f(x) = \frac{\ln 2x}{x^2}$$

is the quotient of two functions of $x$, we'll need to use the quotient rule to differentiate $f(x)$.

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{\left[h(x)\right]^2}$$

In this case, let $g(x) = \ln 2x$ and $h(x) = x^2$. In order to use the quotient rule formula above, we'll need to find the derivatives $g'(x)$ and $h'(x)$. Note that finding $g'(x)$ will require the chain rule, since $g(x)$ is a composite function.

$$g(x) = \ln 2x$$

$$g'(x) = \frac{1}{2x} \cdot \frac{d}{dx} 2x = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

$$h(x) = x^2$$

$$h'(x) = 2x$$

Now, we can use the information we've found to substitute into our original formula to find $f'(x)$.

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$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{\left[h(x)\right]^2}$$

$$f'(x) = \frac{\frac{1}{x} \cdot x^2 - \ln 2x \cdot 2x}{\left(x^2\right)^2}$$

$$f'(x) = \frac{x - 2x \ln 2x}{x^4} = \frac{1 - 2 \ln 2x}{x^3}$$

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